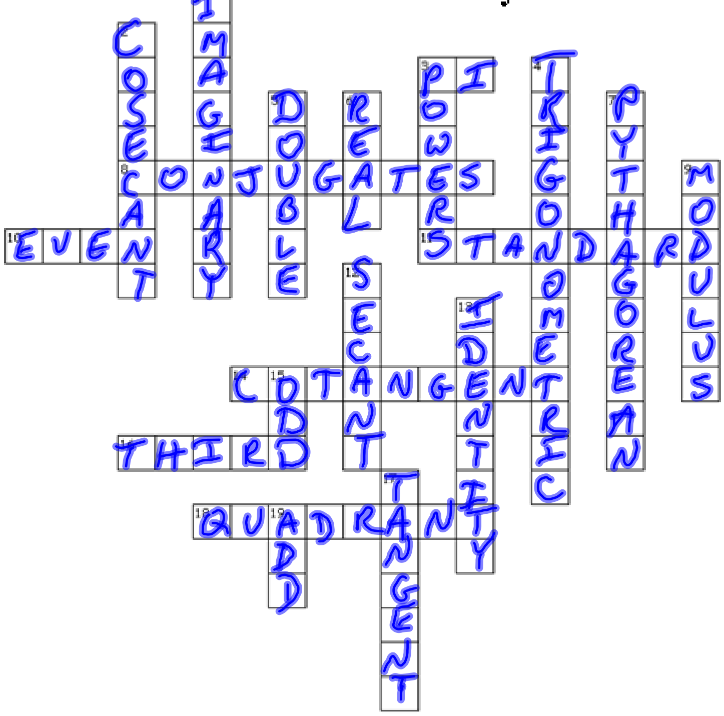
Math 4 Honors
Unit 4 Test Review

Name ______________________________
Date __________________________

Unit 4 Vocabulary



Across

3. The period of the tangent function is ________ radians.
8. Complex numbers z & $\bar{z}$ are _______ of each other.
10. If $f(-x) = f(x)$, then f is a(n) _______ function.
11. $6 + 10i$ is a complex number written in _______ form.
14. _______ is the reciprocal of tangent.
16. The complex number $-3 - 8i$ lies in the ________ quadrant.
18. When converting a complex number in rectangular form to polar form, you must make sure the argument, θ, is in the correct _______.

Down

1. In the complex plane, the vertical axis is the _______ axis.
2. _______ is the reciprocal of sine.
3. DeMoivre's Theorem enables us to calculate _______ of complex numbers.
4. The number $10(\cos 150° + i\sin 150°)$ is written in _______ form.
5. $\cos(2\theta) = 2\cos^2\theta - 1$ is a(n) _______ angle formula.
6. In the complex plane, the horizontal axis is the _______ axis.
7. $\sin^2\theta + \cos^2\theta = 1$ is a(n) _______ identity.
9. The absolute value of a complex number is also called the _______.
12. _______ is the reciprocal of cosine.
13. An equation that is true for all values of the variable (as defined by the domain).
15. If $f(-x) = -f(x)$, then f is a(n) _______ function.
17. To calculate the argument of a complex number, you must take the inverse _______ of b/a.
19. When multiplying two complex numbers in trig. form, you multiply the moduli and _______ the arguments.

1. Let $z = -3\sqrt{3} - 3i$. III

a. Write z in **rectangular** form. $(-3\sqrt{3}, -3)$

b. Find $|z|$.

$r = \sqrt{27+9} = \sqrt{36} = 6$

c. Write z in **polar** form. Keep θ in degrees.

$\tan\theta = \frac{-3}{-3\sqrt{3}} = \frac{1}{\sqrt{3}}$ $\theta' = 30°$ $\theta = 30° + 180° = 210°$

$[6, 210°]$

Unit 4 Test Info:

1. You will be able to use your calculator.
2. You will be given a formula sheet that has the Pythagorean, Sum & Difference and Double Angle Identities. It will also have the Complex n^{th} Roots Theorem formula.
3. You must have the Unit Circle and all other formulas memorized.

2. Express $[4, 315°]$ in *exact* **standard** form.

$a = 4\cos 315°$
$= 4 \cdot \frac{\sqrt{2}}{2}$
$= 2\sqrt{2}$

$b = 4\sin 315°$
$= 4 \cdot -\frac{\sqrt{2}}{2}$
$= -2\sqrt{2}$

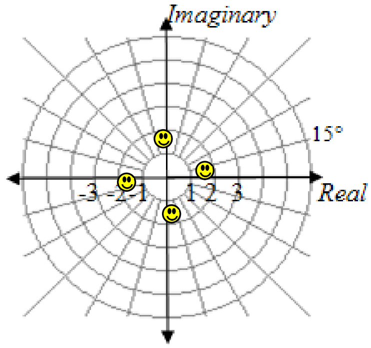
$\boxed{2\sqrt{2} - 2i\sqrt{2}}$

3. Find all fourth roots of $\frac{9\sqrt{3}}{2}+\frac{9}{2}i$. Write your answers in polar form. Keep θ in degrees.

Graph the results below.

$r = \sqrt{\frac{243}{4}+\frac{81}{4}} = \sqrt{81} = 9$

$\theta = \frac{1}{\sqrt{3}} = 30°$



$K=0$ $\left[\sqrt[4]{9}, \frac{30+360\cdot 0}{4}\right]$

$= [\sqrt{3}, 7.5°]$

$K=1$ $[\sqrt{3}, 97.5°]$

$K=2$ $[\sqrt{3}, 187.5°]$

$K=3$ $[\sqrt{3}, 277.5°]$

What type of figure is created by connecting the four roots?

$\frac{360}{4} = 90°$ Square

4. Evaluate each. $0° \le \theta < 360°$ Write your exact answers in the form they started in.

a. z^9 if $z = -1 + \sqrt{3}i$ II

$r = 2 \quad \theta = 120°$

$[2^9, 120° \cdot 9] = [512, 1080°]$

$a = 512 \cos 1080° = 512$

$b = 512 \sin 1080° = 0$

$512 + 0i =$

$\boxed{512}$

b. $\left[3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]^4$

$= 3^4\left(\cos\frac{3\pi}{4}\cdot 4 + i\sin\frac{3\pi}{4}\cdot 4\right)$

$= 81\left(\cos 3\pi + i\sin 3\pi\right)$

$= 81\left(\cos\pi + i\sin\pi\right)$

c. $w \cdot z$ and $\frac{w}{z}$ if $w = [2, 150°]$ and $z = [3, 60°]$

$w \cdot z = [6, 210°]$

$\frac{w}{z} = [\frac{2}{3}, 90°]$

5. Suppose that $\sin\theta = \frac{-4}{7}$ and $\tan\theta < 0$.

a. Find the exact values of each of the following.

i. $\cos\theta = \frac{\sqrt{33}}{7}$

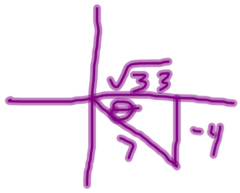
ii. $\tan\theta = \frac{-4}{\sqrt{33}}$

iii. $\csc\theta = -\frac{7}{4}$

iv. $\sec\theta = \frac{7}{\sqrt{33}}$

v. $\cot\theta = \frac{\sqrt{33}}{-4}$

vi. $\sin(2\theta) = \frac{-8\sqrt{33}}{49}$



b. Identify a possible approximate positive degree measure for θ. Show your work.

$\tan\theta = \frac{-4}{\sqrt{33}}$

$\theta' = -34.84$

$\theta = 325.15°$

Verify the identities.

6. $\sin\left(\frac{\pi}{6}+x\right)+\sin\left(\frac{\pi}{6}-x\right)=\cos x$

$\sin\frac{\pi}{6}\cos x+\cancel{\sin x\cos\frac{\pi}{6}}+\sin\frac{\pi}{6}\cos x-\cancel{\sin x\cos\frac{\pi}{6}}$

$\frac{1}{2}\cos x+\frac{1}{2}\cos x=$

$\checkmark\cos x=$

7. $\sec^4 x-\sec^2 x=\tan^4 x+\tan^2 x$

$\sec^2 x(\sec^2 x-1)=$

$(1+\tan^2 x)(\tan^2 x+1-1)=$

$(1+\tan^2 x)(\tan^2 x)=$

$\checkmark\tan^4 x+\tan^2 x=$

Solve for primary values.

8. $\sin x \tan x + \sqrt{3} \sin x = 0$

$\sin x(\tan x + \sqrt{3}) = 0$

$\sin x = 0 \qquad \tan x = -\sqrt{3}$

$\boxed{x = 0, \pi, 2\pi \qquad x = \frac{2\pi}{3}, \frac{5\pi}{3}}$

Find all exact complex solutions.

(Write your answers in standard form.)

9. $x^3 = -8i$

$r = 8 \qquad \theta = 270°$

$K=0 \quad \left[\sqrt[3]{8}, \frac{270 + 360 \cdot 0}{3}\right]$

$= [2, 90°] = 2i$

$K=1 \quad = [2, 210°] = -\sqrt{3} - i$

$K=2 \quad = [2, 330°] = \sqrt{3} - i$

Solve the following problem. Round your answers to the nearest 100th. (degree mode)

10. Because of ocean tides, the depth of the River Thames in London varies as a function of time that involves the sine. Suppose the depth d in meters as a function of t, the hour of the day, is modeled by

$$d(t) = 3\sin\left(\frac{\pi}{6}(t-4)\right) + 8,$$

where $t = 0$ corresponds to midnight, or 12:00 A.M.

30°

a. Predict the depth of the river at 2:00 P.M.
SHOW YOUR WORK.

$t = 14$

$d(14) = 3\sin\left(30^\circ(14-4)\right) + 8$

≈ 5.40 ft.

b. At approximately what times is the depth 10 m?
SHOW YOUR WORK.

$3\sin(30^\circ t - 120^\circ) + 8 = 10$

Let $u = 30t - 120$

$3\sin u + 8 = 10$

$3\sin u = 2$

$\sin u = \frac{2}{3}$

$180 - 41.8^\circ$

$u \approx 41.8^\circ$ or 138.19°

$30t - 120 = 41.8 + 360n$

$\frac{30t}{30} = \frac{161.8}{30} + \frac{360n}{30}$

$t = 5.4 + 12n$

$t = 5.4,\ 17.4$

$30t - 120 = 138.19 + 360n$

$\frac{30t}{30} = \frac{258.19}{30} + \frac{360n}{30}$

$t = 8.61 + 12n$

$t = 8.61,\ 20.61$

5:23am, 8:36am,

5:23pm, 8:36pm